South Asian Geometry and the Durga Temple, Aihole

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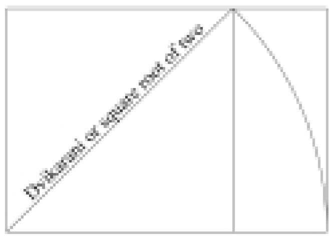
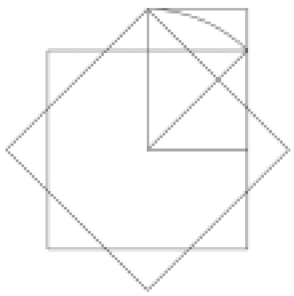
Embedded within the design of South Asian temples are multiple layers of cultural knowledge. Iconographic programmes to guide practitioners, messages of kingship and dynastic power, conceptions of the universe, and the dynamic processes of manifestation and dissolution, could all have a place in a temple's design. Knowledge of geometry, both as an abstract science and part of a symbolic language, can also be found embedded in the design of a temple. How this came to be done can be difficult to determine. Most published plans lack the scale and precision to be of much use to the researcher, but when accurate measurements are available, the use of computer aided design tools can reveal a previously undiscovered, and perhaps unexpected, degree of geometric sophistication.

The use of the *vastupurusha mandala*, a ritual grid typically consisting of 64 or 81 squares, and often thought to be rooted in the geometry of Vedic altars, was once taken as an archetypal element underlying all Hindu temples (Kramrisch 1946, p. 33). This notion has recently been reassessed (Bafna 2000, pp. 26-49). The primary evidence for its use came from the 5th century *Brihatsamhita*, an encyclopedic work whose primary topic is astrology. However, neither Vedic texts nor architectural works like the *Mayamatam* or *Manasara* make any reference to the *vastupurusha mandala*. It may nevertheless be argued that a ritual grid was used in the design of some temples, but its use was never universal. Furthermore, where evidence of its use is strong, its presence does not preclude the use of other geometric devices involving points of intersection between circles, squares, triangles, and patterns of radiating lines (Volwahsen 1969, pp. 52-53).

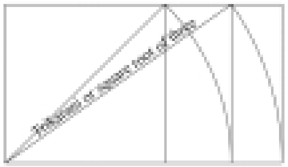
The earliest Indian texts to deal with geometry directly are the *Sulbasutras*. Variously dated from as early as 800 B.C. to as late as the 4th century B.C. (Amma 1979, pp. 14-15), the *Sulbasutras* cover the geometric formulae necessary to construct *vedis*, or altars and *agnis*, or fire pits. While the *Sulbasutras* vary in outline and depth, they typically progress in successive chapters from abstract geometric principles to the altars necessary for specific rituals, with detailed descriptions of the shapes and sizes of the bricks required for altar construction. They show how to establish an east-west axis and bisect it with a north-south line, how to manipulate areas such as converting a given rectangle into a square and vice-versa, how to add and subtract squares of different areas, how to convert a square into a circle and vice-versa, and how to form various trapezoids out of squares or rectangles. What we know as the Pythagorean Theorem was understood by the *Sulbasutras*. They also understood the geometric significance of the square roots of two and three: if a square is built on the diagonal of another square, the second will be twice the area of the first. They therefore named the diagonal of a square the *dvikarani*, or "double maker." Moreover, they knew that, if a rectangle is formed with the proportion of one to the square root of two, its diagonal will be the *trikarani*, or "triple maker." That is, a new square built on this diagonal has an area three times that of the original square. The *dvikarani* and *trikarani* are of even greater interest because their proportions are inherent in two common star polygons — the eight-pointed star (*ad quadratum*) and the six-pointed star (*ad triangulum*) (Figs. 1-4). To my knowledge these two stars and their proportions are not mentioned in the *Sulbasutras*. However it seems likely that the Early Western Chalukya builders, who did use these stars, were aware of at least some of the *sulbasutras*, and their system of geometry should be understood as growing out of, and building upon, the earlier Vedic tradition of altar building.



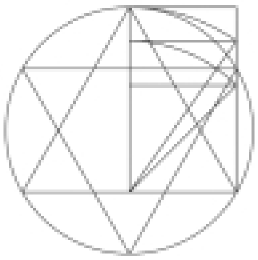
1. A rectangle in the proportion of 1:√2.



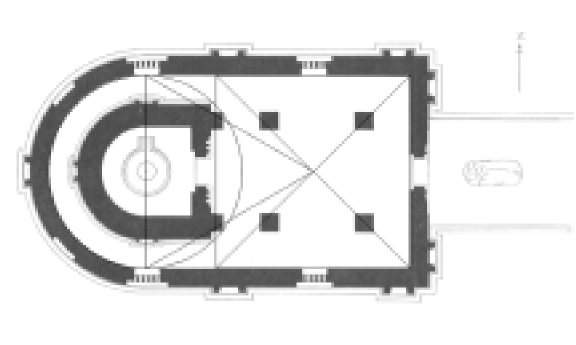
2. The proportion of 1:√2 in an *Ad Quadratum* construction.



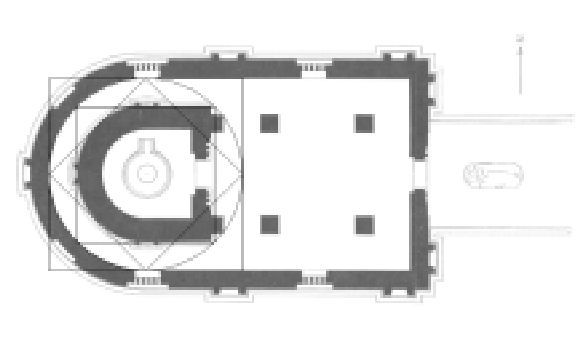
3. Rectangle in the proportion of 1:√3.



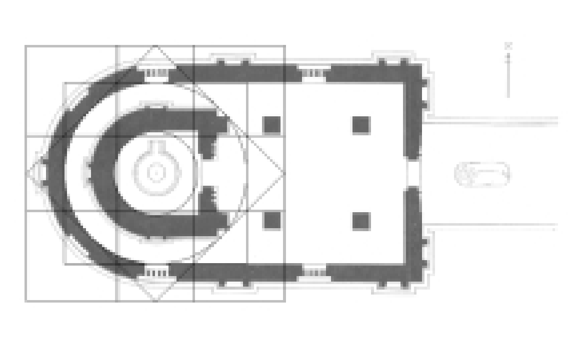
4. The proportion of 1:√3 in an *Ad Triangulum* construction.



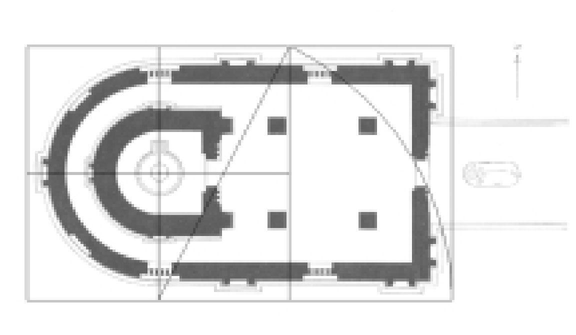
5. Proportions of Chika Mahakut (after a plan by Bolon).



6. The use of *ad quadratum* in Chika Mahakut (after a plan by Bolon)



7. method for locating the garbhagriha at Chika Mahakut (on a plan by Bolon).

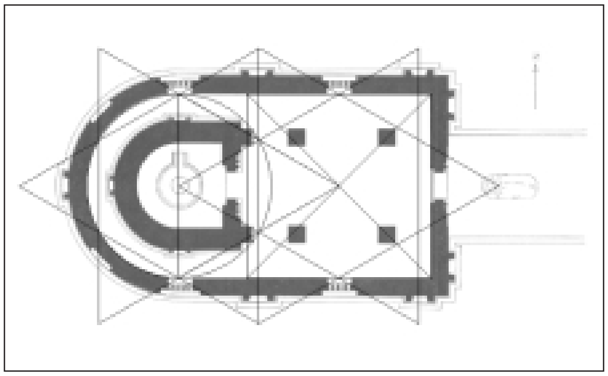


8. Chika Makahut by a recangle in the proportion of 1:Φ (the so-called golden section), on a plan by Bolon.

Recently, through the use of computer models of the Durga temple of Aihole and related temples, I have come across a type of geometry that appears to be based primarily on a pattern of interconnected star polygons (five, six, eight and twelve-pointed stars), concentric circles, and radiating lines which were used to fix the proportional relationships between the various walls of the temple. How widely this type of geometry was applied, or how it might relate to the use of *yantras* in meditation is unknown. When a *yantra* of concentric circles and radiating lines is used to organize pilgrimage routes in Varanasi (Singh 1997, pp. 197-199), or eight pointed stars are used to organize goddess shrines in Nepal (Levy 1990, pp. 153-155), the geometry is conceptual and not actually built into the physical environment. Similarly, when *yantras* are part of a drawn or painted mandala (Khanna 1979, pp. 45-52), the symbolic or conceptual nature of the form, rather than its precise geometric properties, seems to be the primary consideration. In contrast, the way Early Western Chalukya architects of the Durga temple use yantras, or star polygons, suggests a body of geometric knowledge unlike other traditions of sacred geometry, either in the West or elsewhere in India.

Geometry and the apsidal temple at Chika Mahakut

Before analyzing the more complex Durga temple, it is worth taking a brief look at the much smaller, much simpler, apsidal Early Western Chalukya temple from nearby Chika Mahakut. Because I have only Bolon's plan from the *Encyclopaedia of Indian Temple Architecture* available (Meister ed. 1983, p. 63), and not actual measurements, I will make only a few general observations about the overall proportions.



9. A triangular grid on Chika Mahakut (on a plan by Bolon).

First, fit a square into the *mandapa* (hall), and a circle along the inner face of the ambulatory's outer wall as shown (Fig. 5). Next, fit an equilateral triangle into the circle of the ambulatory so that its base corresponds to the diameter of the circle. The apex of the triangle will then fall in the centre of the mandapa's square. What diagram the builders actually used to arrive at this proportion is not known. They may have used a six pointed star, a simple triangle as shown, or some larger, as yet undetermined, construction. The only thing we can be sure of is that if the width of the hall (and diameter of the ambulatory) equals 2, then the distance between the centre of the circle and the centre of the square equals $\sqrt{3}$. Next, if an *ad quadratum* is constructed in the circle of the ambulatory, it appears to locate the outer walls of the garbhagriha (Fig. 6). The inner walls of the garbhagriha are a little trickier. Various combinations of *ad quadratum* or *ad triangulum*, singularly or nested one inside the other, do not get very close. However, if another *ad quadratum* is constructed on the outside the previous one, we can locate a square that falls outside the limits of the temple, which seems relevant. The central one third of this new square locates the inner walls of the garbhagriha (Fig. 7). Interestingly (and this may be purely coincidental), if one divides this new square in half, and takes the diagonal of one half to draw an arc to the west, one can construct a rectangle in the proportion of 1:Φ (the so-called golden section), which appears to frame, but not actually touch, the temple walls (Fig.8).

One more construction of note at Chika Mahakut involves the apparent use of a triangular grid. This is built off the first triangle we located between the circle of the ambulatory and the centre of the *mandapa* and in effect results in two interlocking six-pointed stars (Fig. 9). This grid appears to locate the centre-lines of both the windows and exterior niches on the north and south sides of the temple. Again, as before, we are probably looking at an effect of the construction the original architects used rather than the construction itself. And, of course, I am assuming that Bolon's plan is accurate. Actual measurements would need to be obtained to verify my observations.

10. Durga Temple of Aihole from the southeast. (Photograph by the author.)

11. Durga Temple of Aihole from the northwest. (Photograph by the author).

Geometry and the Durga Temple, Aihole

The so-called Durga temple at Aihole (Fig. 10-11) has been well studied over the years, and has even been the subject of a recent historiographical study (Tartakov 1997). Its name does not come from being dedicated to the goddess Durga, but from its proximity to a fort or durga. There is no central shrine image and many of the sculptures in the ambulatory are missing. However, based on inscriptions on an adjacent gate and some iconographic details, it is now generally considered to be an eighth-century, Early Western Chalukya temple dedicated to the god Aditya (Tartakov, 1997, p. 42). It takes the form of an apsidal temple with inner and outer ambulatories — a form early researchers con-sidered a derivative of Buddhist *chaitya* halls, but is now generally recognized as a traditional Brahmanical form. The northern-style sikhara looks a little incongruous, and some have speculated that it was a later addition, but it is now recognized as well within Early Western Chalukyan stylistic limits (Huntington 1985 p. 332).

Karnataka received influences from both the northern and southern architectural styles of India, and the Early Western Chalukyas often built in the two styles side by side. The Durga temple shows the influences of both. Around the outer ambulatory are eleven sculpture niches (now mostly empty) and six windows, each of which shows a unique architectural composition. These are mostly composed of southern elements, but with some northern, and a few entirely original elements as well.

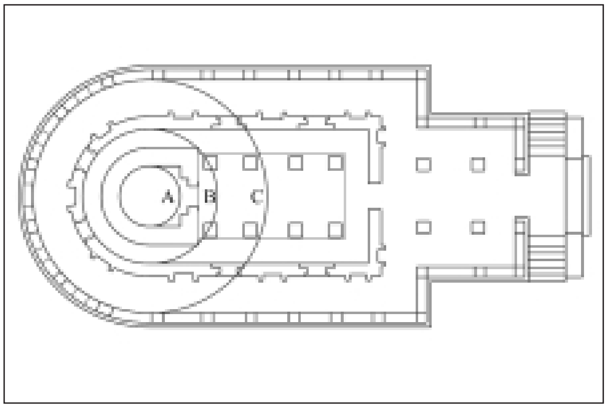
After a visit to India in 2001 I began considering the proportions of the temple, but discovered that the best published plans, those of George Michell (Michell 1975, plate 24), showed some puzzling eccentricities. For example, the plan as published (and blown up on a photocopy machine) showed the south wall of the *garghagriha* to be 8 to 10 centimetres thicker than the north. Of course, Michell's plans were intended to show the general architectural form, and so did not provide the necessary level of precision. Graciously, Dr. Michell sent me copies of his original field measurements taken more than 30 years earlier. With these measurements I redrew the plan using a CAD program, which not only cleared up the irregularities of the published plan, but helped me to discover that the builders of the temple used some simple geometric proportions with an astonishing degree of precision — particularly considering how eroded and imprecise much of the stonework appears. The one area where my study of the geometry breaks down is on the porch. Photographs of the floor of the porch (Fig. 12, 13) show cracks from settling which may account for this, or there may well be something else going on which I have been unable to solve. In any event, I will for the most part limit my comments to the main body of the temple.

12. Photograph by Gary Tartakov showing cracks in porch floor before the niche to the left of the entrance.

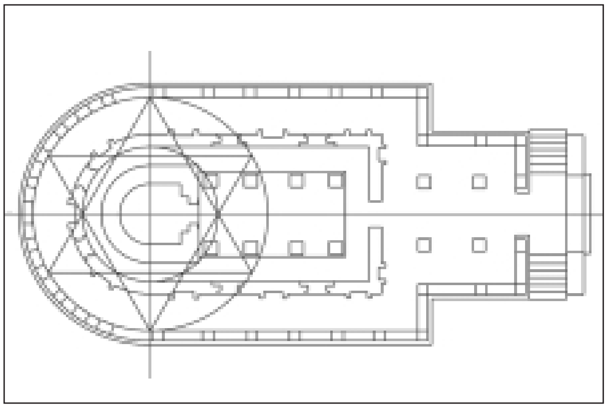
13. Photograph by the author showing mortar filled cracks in the porch floor before the niche to the right of the entrance.

The Durga temple is larger and more complicated than Chika Mahakut but demonstrates similar kinds of relationships. First, it will be useful to start with some limits. It is my suspicion that the temple was actually built with clues to decoding its geometry — perhaps so that its ritual precision could be demonstrated or so that future generations of architects and students would have clues when puzzling it out for themselves. Thus, some of the most critical dimensions needed to begin decoding its underlying geometry are also some of the easiest to measure. Examination shows that some of the Durga temple's walls are complicated with base moldings, niche projections, and recesses while others run straight to the floor, unencumbered by moldings or niches. It is just those wall surfaces without moldings that are the most critical to the temple's proportions, and the easiest to measure. They are: the interior walls of the inner sanctum (and the diameter of the circle forming its western half), the interior width of the main hall (and the diameter of the circle forming the ambulatory continuous with the hall), and the width of the outer ambulatory (and the diameter of the circle of the outer ambulatory). Once one has determined the diameter of these three circles (identified in Fig. 14 as A, B, and C), one can construct the diagram to which the rest of the temple is tied.

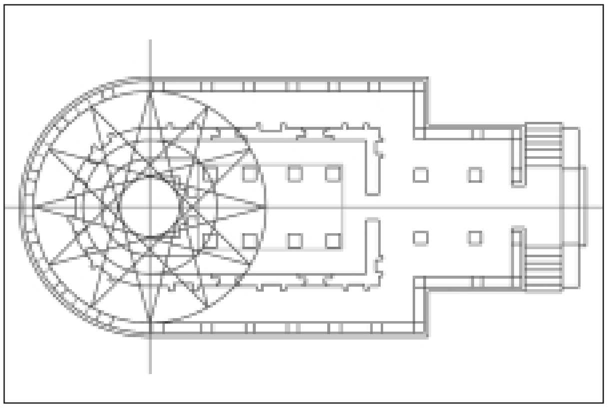
The easiest proportion to locate is between inner and outer ambulatory circles (Fig. 15). If one constructs a six-pointed star, or *ad triangulum*, in the outer circle (diameter 1064 cm), then the point where the two triangles overlap locates circle B (diameter 614 cm). This fits so perfectly that the margin of error is less than one centimetre. Similarly, if one places a twelve-pointed star inside the circle of the outer ambulatory, it will bound the circle of the inner sanctum with a margin of error of about one third of a centimetre (Fig. 16). It should be noted that Michell's measurements were only made to the nearest centimetre, so these margins of error are less than what can be detected with the current set of measurements.



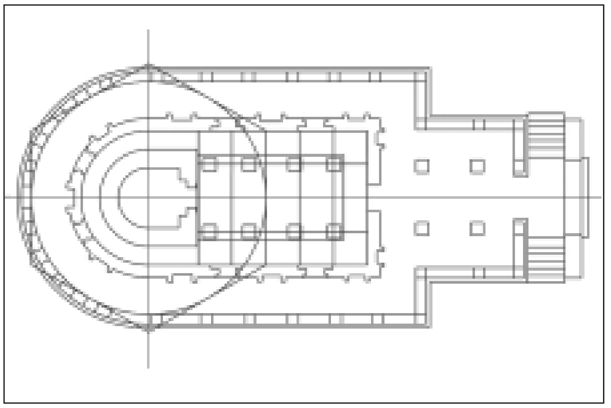
14. The three key circles of the Durga temple.



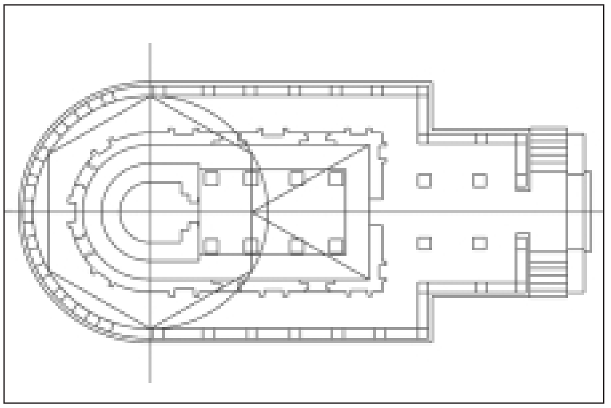
15. *Ad triangulum* between the inner and outer ambulatories.



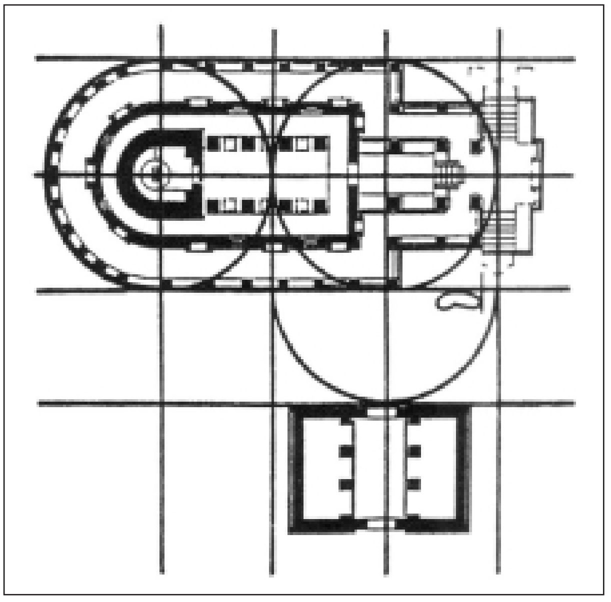
16. A twelve-pointed star placed in the circle of the outer ambulatory locates the circle of the inner sanctum.



17. Using a hexagon around the circle of the outer ambulatory to locate the 4x5 grid of the main hall.



18. An equilateral triangle fitted between the main hall's inner east wall and a hexagon inside the circle of the outer ambulatory.



19. A diagram of the Durga temple's overall proportions, and its relationship to the gateway, as proposed by Tartakov.

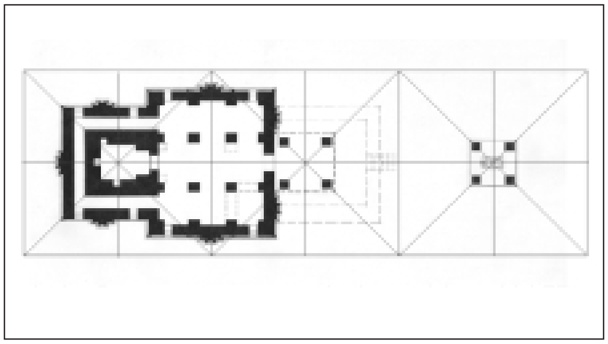


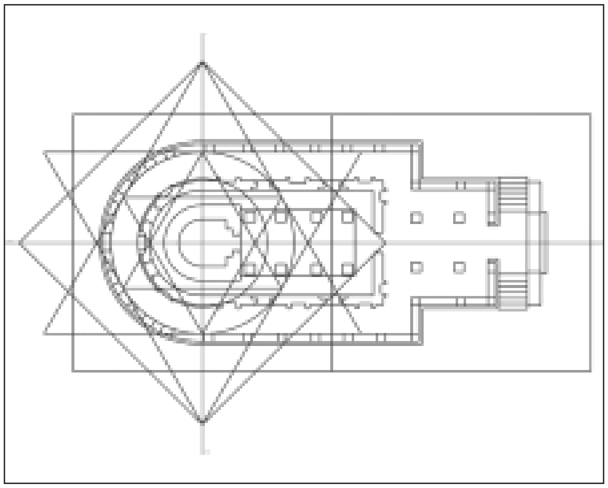
Figure 20. The site plan of Mahakuteshvara proportioned by a triple square (diagram by the author after a plan by Michell).

Next, if one wraps a hexagon around the outer ambulatory circle, then its eastern face divides the hall at a point 3/5 from the entrance. In fact, it turns out that the hall as a whole is in the proportion of 4x5 (Fig. 17). This is not quite as precise. Michell's measurements show some slight settling in the hall. Its width at the entrance is 614 cm. At its midpoint it is 618 cm. And in front of the inner sanctum it measures 614 cm again. The overall length of the hall is 771 cm so a perfect 4x5 rectangular grid would require a width of 616.8 cm — a margin of error not entirely conclusive, but not one to dismiss out of hand. It should be pointed out that this grid cannot be extended to fit the overall structure (or at least, if it is extended, it does not correspond to anything else).

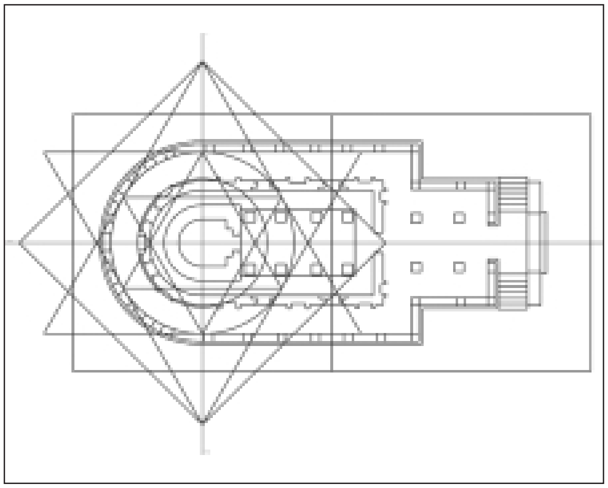
There is another method to locate the depth of the hall that is even more accurate than the 4x5 grid. If one draws a hexagon inside the outer ambulatory circle, then an equilateral triangle with sides of 614 cm each can be fitted with its apex on the centre of the eastern face of the hexagon, and its base set on the inner eastern wall of the hall, as shown (with a margin of error less than 1 cm) (Fig. 18). It seems unlikely that the builders actually used the constructions shown in Figures 17-18, whereas they probably did use those shown in Figures 15 and 16. Rather, I believe, they used some construction based on triangulation that produced the effects we can observe.

The next question concerns the temple's overall proportions. Gary Tartakov has published a plan (Fig. 19) showing a grid and two side-by-side circles with their diameters set to encompass base (*adisthana*) moldings of the temple (Tartakov 1997, p 63). If this is the case, then the eastern edge of the eastern circle falls 13 cm to the west of the centre-line of the stair — a rather inelegant solution even if we consider that the porch and stairway may have settled a few centimetres over time. As for the relationship between the temple and its gateway to the south, little can be said because no measurements are available. Nevertheless, it seems odd to place it half a circle away, and at the same time out of alignment with the module the circles establish. Tartakov's diagram starts with the assumption that the outer limits of the geometry governing the temple must conform to the limits of its base. However, it is quite reasonable to assume that the overall governing diagram extends over the larger building site, and that the geometry of the building is then tied to the larger diagram.

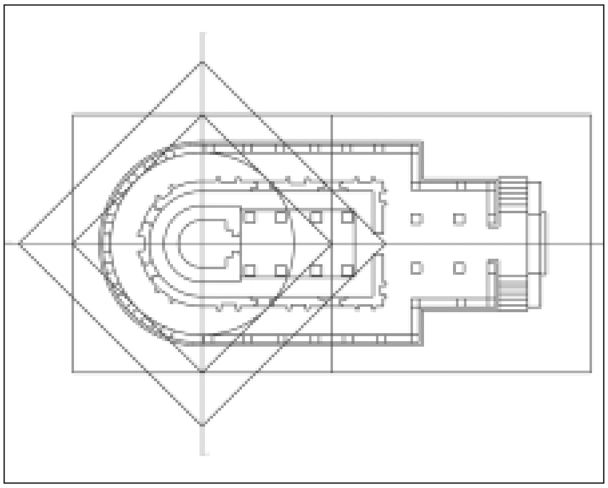
Consider briefly the plan of another Early Western Chalukya temple, the Mahakutesvara temple at nearby Mahakut (Fig. 20). While no actual measurements of the temple are available, and analysis must rely on a photocopy of Michell's plan, it appears that the inner sanctum, the porch, and the small Nandi *mandapa* (pavilion for Shiva's bull) are all tied to a three-square grid over the site. Similarly, the Durga temple appears to



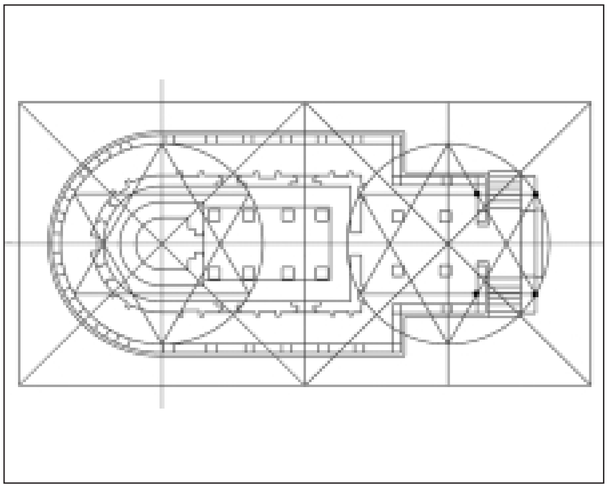
21. *Ad triangulum* expanded on the outside of the outer ambulatory circle.



22. *Ad quadratum* matched to the extreme N-S points of *ad triangulum*, and a copy of one square moved east to form a double square over the whole site.



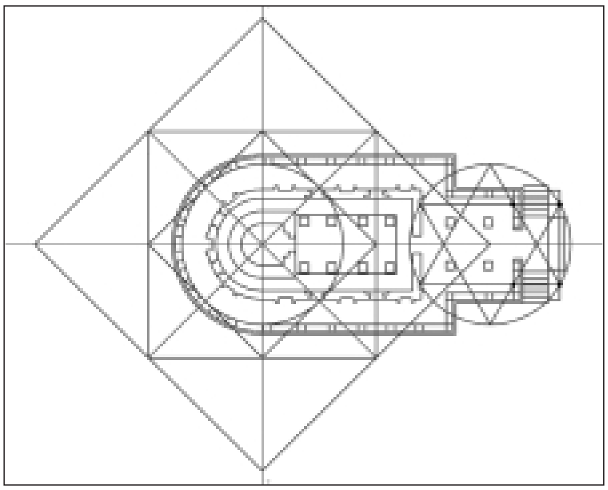
23. Arriving at the double square by placing a square around the outer ambulatory circle, and then building out with *ad quadratum* and continuing as before.



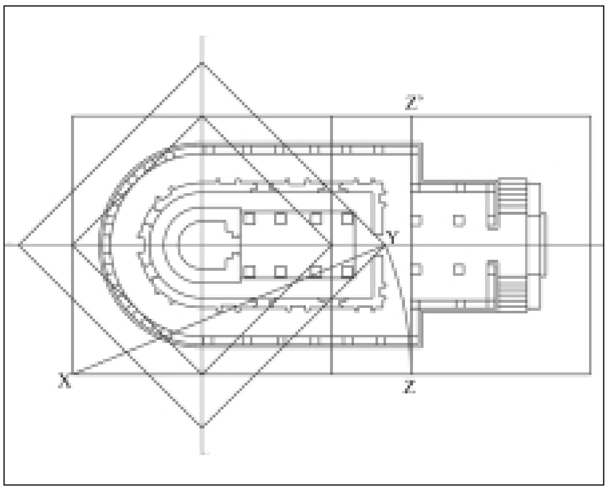
24. The circle and *ad triangulum* from the outer amubulatory mirrored in a square over the east end to locate several key points on the porch.

fit into a double square which, when located, can be used to fix other parts of the temple. This double square can be located in two ways, or located by one and confirmed by another. The first is to create an expansion of the *ad triangulum* motif in the circle of the outer ambulatory, this time on the outside of the circle as shown (Fig. 21). Now draw a large *ad quadratum* over the large *ad triangulum* so that their extreme northern and southern points touch (Fig. 22). Next move a copy of the square that is aligned with the temple, so that it frames the porch. Alternatively, one can place a square diamond fashioned around circle C of the ambulatory, and then expand this square outward in a 1:√2 proportion to arrive at a larger square aligned with the temple. Then move a copy of this square over the porch as before (Fig. 23). This new square is not actually centred on the porch, but from it we can locate several points related to the depth of the porch. Draw a circle and *ad triangulum* the same diameter as the outer ambulatory circle, but this time centred on the eastern square (Fig. 24). The points where the two triangles overlap locate the inner eastern face of the porch (within half a centimetre) and the eastern most points of the triangles fall on one of the lines defining the outer eastern end of the temple (also within half a centimetre). It might be pointed out that the eastern circle and star could just as easily be positioned by other means, such as a pattern of expanding squares (Fig. 25), but the double square has a certain compact simplicity.

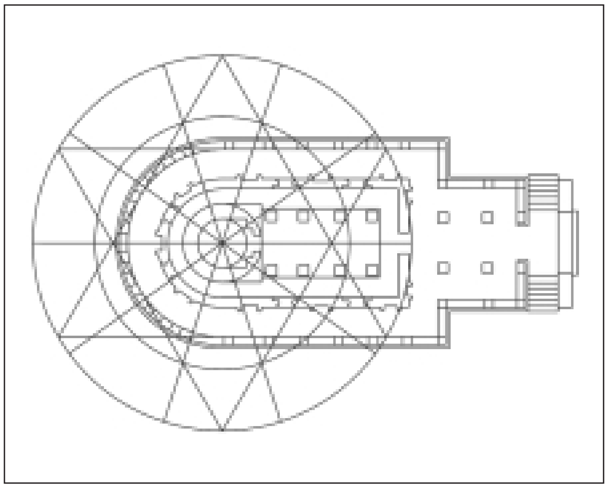
From here it should be noted that the outer ambulatory is not a uniform depth. Along the north, south, and west, the ambulatory is 5 cm deeper than



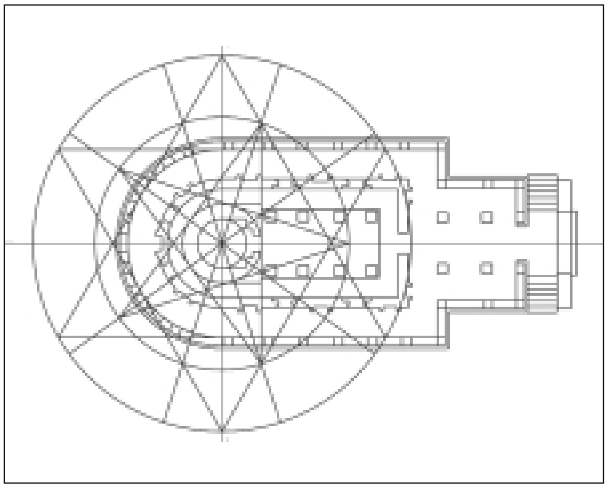
25. Alternate method for locating the eastern circle and star.



26. Construction showing how to locate the east end of the ambulatory from the large *ad quadratum*.



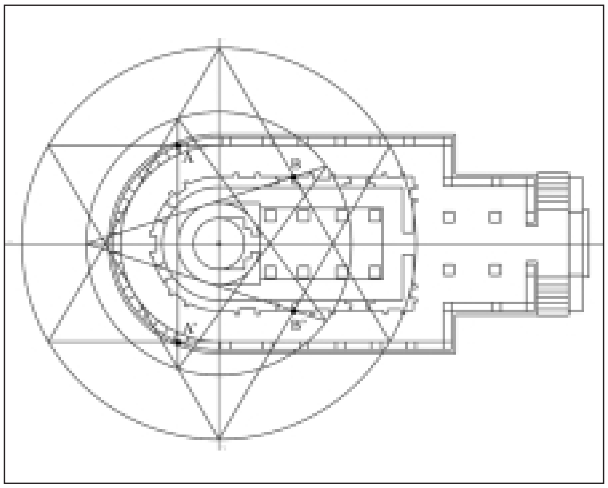
27. A new circle drawn where ten radiating lines intersect with a large *ad triangulum*.



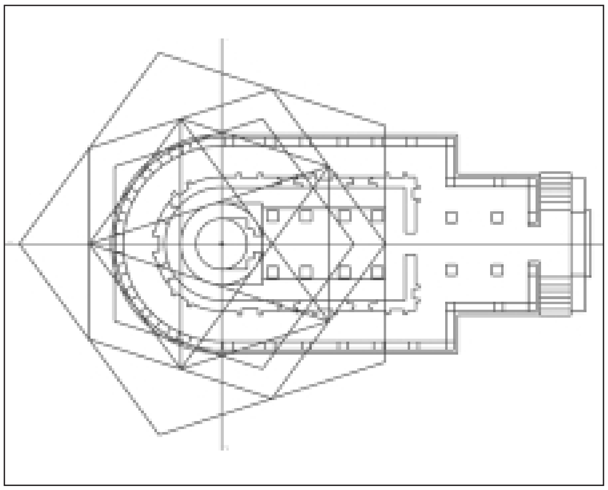
28. A pentagram drawn inside the new circle establised in Figure. 27. The inner edges of the pentagram locate the east face and outer walls of the inner sanctum.

along the east, so we need a method to locate that wall other than by simply wrapping a uniform ambulatory around the hall. This can be done with the use of the large *ad quadratum*, as constructed in Figure 23. First, draw an arc, centred on the southwest corner at X from the eastern point Y down to Z as shown (Fig. 26). Then draw a line north to Z′. The line Z-Z′ now falls along the inner face of the eastern wall of the outer ambulatory. Remarkably, the margin of error here is still less than one centimetre. This is a proportion that I have never seen employed in sacred architecture before, East or West, although its use here seems very natural.

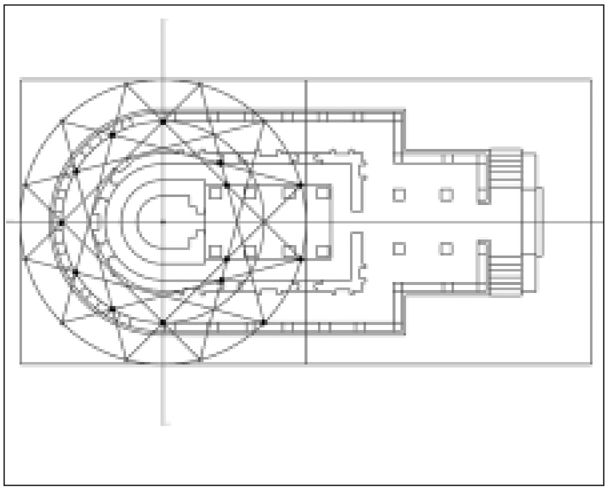
There is at least one more significant feature to the main body of the temple worth calling attention to: the use of pentagrams to locate some of the wall surfaces with base moldings and niches. First, draw a circle around the large six-pointed star of the temple (established in Fig. 21) and then divide the circumference into ten segments with radiating lines coming out from the centre. At the point where the radiating lines intersect the star, draw another circle (Fig. 27). Within this new circle draw two pentagrams, the first with one of its points centred in the mandapa, as shown as shown in Figure 28. The inner pentagon of this pentagram locates the east face and curved outer walls of the inner sanctum (not including the base molding) with as great a degree of precision as we have seen anywhere else – a margin of error less than one centimetre. Next, rotate the pentagram 180 degrees, and note two pairs of points where the pentagram intersects the six-pointed star (points A, A′ and B, B′ in Fig. 29). The first pair (A, A′) fall less than two centimetres inside the circle defining outer



29. When the previous pentagram is rotated 180˚, points of intersection with the six-pointed star locate two additional walls.



30. A pentagram used to locate the western edge of the raised area inside the *mandapa*.



31. A pattern of four triangles inside the western sqaure generated in figures 22 and 23 showing points of correspondence with the outer ambulatory and the northern and southern edges of the raised area inside the *mandapa*.

wall of the outer ambulatory, on the temple's east end (not counting the base molding), and the second pair (B, B') falls less than one centimetre from the line defining the outer wall of the mandapa (again, not counting the base molding). Finally, enclose this second pentagram in a pentagon, and then expand outward in stages with two additional pentagons (Fig. 30). The eastern edge of the last pentagon falls within a centimetre of the eastern edge of the central section of the mandapa floor that has been raised several centimetres above the level of the surrounding ambulatory. The distance between the northern and southern edges of this raised area and their flanking walls are about 5 cm greater than the eastern edge is from the eastern wall so these edges cannot be located by simply wrapping an equidistant passage around the hall. To locate these edges we return to the diagram of the double square, established in Figures 22 and 23. Draw a circle within the western square and then position four triangles within, as shown (Fig. 30). As might be expected from earlier diagrams, some of the triangles points and intersections fall along the lines of the inner and outer ambulatories. What is perhaps unexpected is that the points of two triangles, and points of intersection between two triangles, fall within half a centimetre from the northern and southern edges of the raised area of the mandapa floor.

As mentioned earlier, the porch appears to have settled so that its columns and overall width are out of alignment with the rest of the temple. If one divides the double square located in Figures 22 and 23 with a simple 4x4 grid it comes close to the outer limits of the porch, but the margins of error are too great to be certain – 3 to 4 centimetres on one side and even more on the other. There are also some eccentricities in column spacing both inside the temple and around the outer ambulatory, as well as subtle irregularities in the size and position of niches, for which I am unable to account. It may be that such elements were positioned by a geometric or mathematical system that I have not been able to solve, were shifted off the main pattern of geometry for unknown reasons, or were deemed less critical, and hence less subject to rigid control.

Conclusions

While it would be satisfying to have the additional validation of a Sanskrit text on temple geometry from the late 7th or early 8th centuries, this is not essential. The Durga temple exists as a well-built primary document on the geometry of the Early Western Chalukyas. Their knowledge of geometry was built into the structure in such a way that anyone with a measuring rod and knowledge of basic geometric diagrams could decode. This type of geometry does not appear to be rooted in Vedic altars (though the builders were likely aware of the *Sulbasutras*), or the *Vastupurusha mandala*. Instead, the geometry appears related to the kind of geometric yantras used in tantric meditation. *Mandalas* incorporating such yantras do not survive from this period, and later *mandalas* that do incorporate such devices do not appear to be concerned with their geometric properties in the same way as the Durga temple. Yet it is quite possible either that the use of geometric yantras in meditation grew out of a body of geometric knowledge developed in this period, or else that the direction taken by this system of geometry was driven by the use of such yantras in meditation.

It bears repeating that while the diagrams presented demonstrate that certain geometric proportions were built into the design of the Durga temple, we do not yet know what larger system of geometry was employed. Coming along some twelve centuries after the fact, this study may have separated out aspects of the geometry that would have seemed alien to the builders. It seems probable that the builders of the Durga temple had a coherent a body of geometric knowledge that was concerned with a wide variety of geometric figures and proportions, and that they were particularly interested in how various star polygons could be related to each other. They may have even been working to develop a plan that incorporated the maximum number of such stars into a single design. It would not be surprising if, on some level, a seven or even a nine-pointed star has been worked into the plan, although I have as yet found no evidence for this. There may also be other simple diagrams or proportions, or some form of number theory in use, whose properties are waiting to be uncovered. It will probably take the study of a dozen more Early Western Chalukya temples before we can come closer to a more complete understanding of the system of geometry underlying them. As for the meanings that their builders attached to the forms, we may never know. There is something almost universal about the use of sacred geometry in traditional sacred architecture around the world, but the symbolic weight given to a six, eight, five or twelve-pointed star may vary, not only by religion, but by region and period as well.

BIBLIOGRAPHIC REFERENCES

Amma, T.A. Sarasvati, 1979, *Geometry in Ancient and Medieval India*, Motilal Banarsidass, New Delhi.

Bafna, Sonit, 2000, "On the Idea of the Mandala as a Governing Device in Indian Architectural Tradition", J*ournal of the Society of Architectual Historians* 56, no 1., pp. 26-49.

Huntington, Susan L. and John C., 1985, *The Art of Ancient India*, Weatherhill, New York.

Khanna, Madhu, 1979, *Yantra: The Tantric Symbol of Cosmic Unity*, Thames and Hudson Ltd, London.

Kramrisch, Stella, 1946, *The Hindu Temple*, University of Calcutta, Calcutta.

Levy, Robert I, 1990, *Mesocosm: Hinduism and the Organization of a Traditional Newar City in Nepal*, University of California Press, Berkeley.

Meister, Michael W., (ed.), 1983, *Encyclopaedia of Indian Temple Architecture: Volume 1, Part 1: South India, Lower Dravidadesa, Later Phase, 200 B.C.-A.D. 1324*, University of Pennsylvania Press, Philadelphia.

Michell, George, 1975, *Early Western Calukyan Temples*, AARP, London.

Singh, Rana P.B., 1997, "Sacred Space and Pilgrimage in Hindu Society: the Case of Varanasi." in *Sacred Places, Sacred Spaces: The Geography of Pilgrimages*, Stoddard, Robert H., and Morinis, Alan, ed. Louisiana State University, Baton Rouge.

Tartakov, Gary Michael, 1997, *The Durga Temple at Aihole: A Historiographical Study*, Delhi, Oxford University Press.

Volwahsen, Andreas, 1969, *Living Architecture: Indian*, Grosset & Dunlap, London.