<u>Geometry and Geometers of Later Islamic Architecture</u>

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History of Art 664

While the role of geometry in much Islamic architectural ornamentation is immediately apparent, its role in plans and elevations is a little less so. It is also unclear, based on the limited information available, what the role between geometer, architect, and craftsmen may have been in different periods of later Islamic architecture. There are a few texts available as well as the monuments themselves, but the texts present somewhat contradictory views and the monuments are open to various interpretations. Adding to the confusion are some scholars who take a sort of collective or universal view of Islamic geometry. For example Issam El- Said, in his book *Islamic Art and Architecture: The System of Geometric Design,*[1] presents a beautiful survey of patterns based on square and hexagonal grids but without distinguishing the particular period or region that employed these particular patterns. Another author, Keith Critchlow, a sort of New-Age, Neo-Platonist geometer and architect, has written works such as *Order in Space: a Design Source Book*[2] that is an unparalleled study of two and three-dimensional geometry, but when writing on Islamic geometry[3] he seems more interested in appropriating it into his own philosophy than placing it in its historical cultural context. The principal objective of this paper will be to piece together a somewhat fragmented view of the use of geometry in its cultural context based on texts, scrolls, and examination of monuments.

What is a geometer?

In the book *Timurid Architecture of Iran and Turan*, Golombek and Wilber write, "the highly skilled architect was known as a *muhandis*, a 'geometer.'"[4] The term *muhandis* is similarly translated by Gulru Necipoglu in *The Topkapi Scroll*[5] although a hyphenated "architect-engineer" is used throughout. In contrast Alpay Ozdural, finds evidence that geometers were distinct from architects and artisans but that these various professionals had meetings he refers to as *conversazione.*[6]

Geometry texts

Ozdural presents quotations by the 16th century Ca'fer Efendi, and the 10th century Abu 'l-Wafa' Al-Buzajani, both of whom complain how the science of geometry is not understood by the architects and craftsmen of their day. In a text titled *The Book on what the Artisan Requires of Geometric Constructions,*[7] Abu 'l-Wafa' describes *conversazione* he attended between geometers and craftsmen. In one of these encounters a geometer is demonstrating a geometric proof for constructing a square equal in area to three smaller squares but the craftsmen are unsatisfied with a result. Abu 'l-Wafa' understood that the craftsmen did not simply need geometric proofs but constructions that are satisfying as ornamental designs. The craftsmen were thinking in terms of physical tiles that can be cut and arranged into patterns so the principal objective of Abu 'l-Wafa''s text is to present designs that are geometrically accurate and visually satisfying. The text first shows how to form a square from two squares, and from five squares, and then shows how the pattern may be expanded radially into an ornamental geometric design (fig 1). By expanding the square formed from 5 squares to an area of 9 squares and then altering the outer almond shapes into three smaller almond shapes, he produced a design which artists could, and did, incorporate into their art (fig 2).

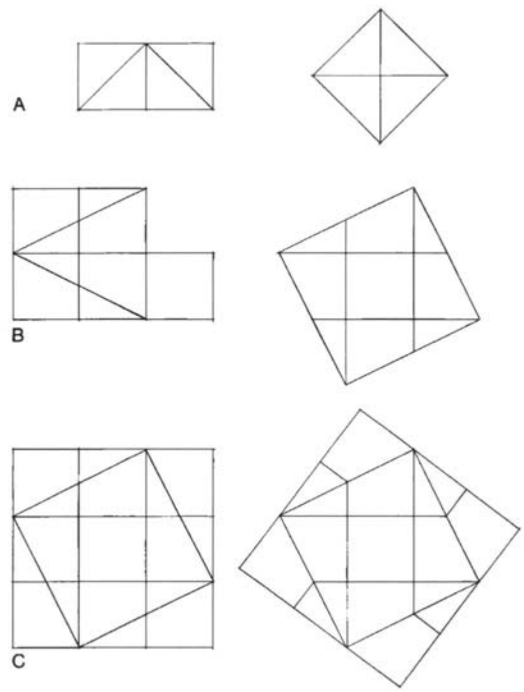


Figure 1. Abu ‘l-Wafa’’s designs for forming squares from 2, 5, and 9 squares.

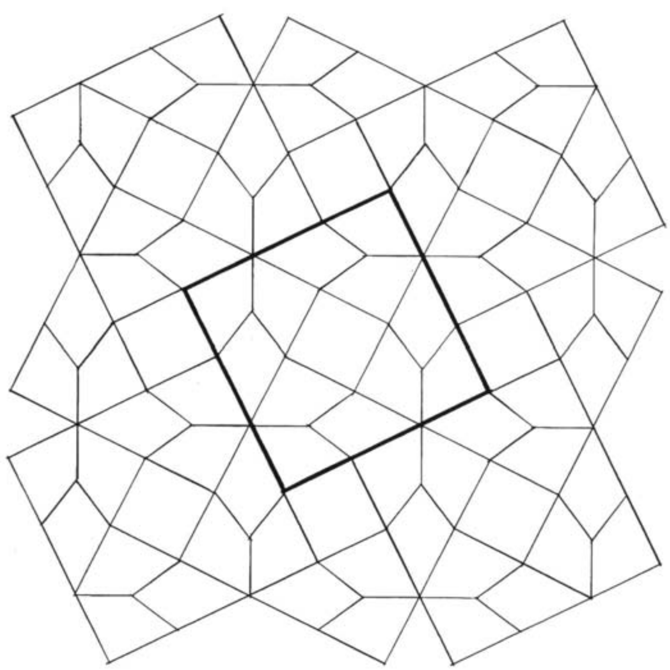


Figure 2. Abu ‘l-Wafa’’s design for a pattern from a square with an area of 9 squares.

In approaching the problem of the square equal in area to three smaller squares Abu ‘l-Wafa’ taught by a series of cut and paste demonstrations. He begins by analyzing some of the craftsmen’s own solutions to the problem and demonstrates by means of geometric proof as to why these are inaccurate (fig 3, 4). He then produced his own solution to the problem although it clearly lacked the decorative potential of the square formed from 5 squares (fig 5). Abu ‘l-Wafa’’s solution to a square of 5 squares seemed to have inspired another interesting pattern. Ozdural notes that around 1074, in an untitled text, “Omar Khayyam described a special right-angled triangle, in which the hypotenuse is equal to the sum of the short side plus a perpendicular to the hypotenuse.”[8] Then around 1300 a special case of Abu ‘l-Wafa’’s solution appeared in the anonymous work, *On Interlocks of Similar or Corresponding Figures,* with rotating triangles of the type described by Omar Khayyam (fig 6).[9]

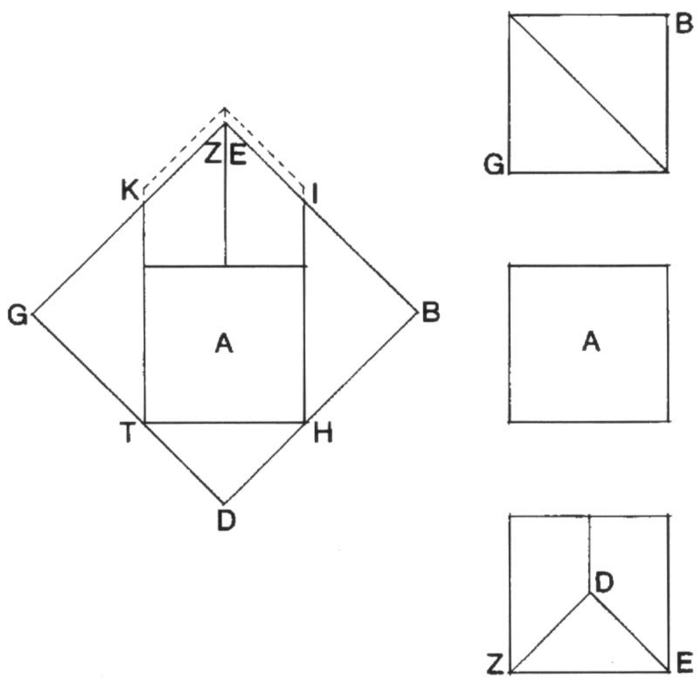


Figure 3. Abu ‘l-Wafa’’s demonstration disproving a solution proposed by a craftsman.

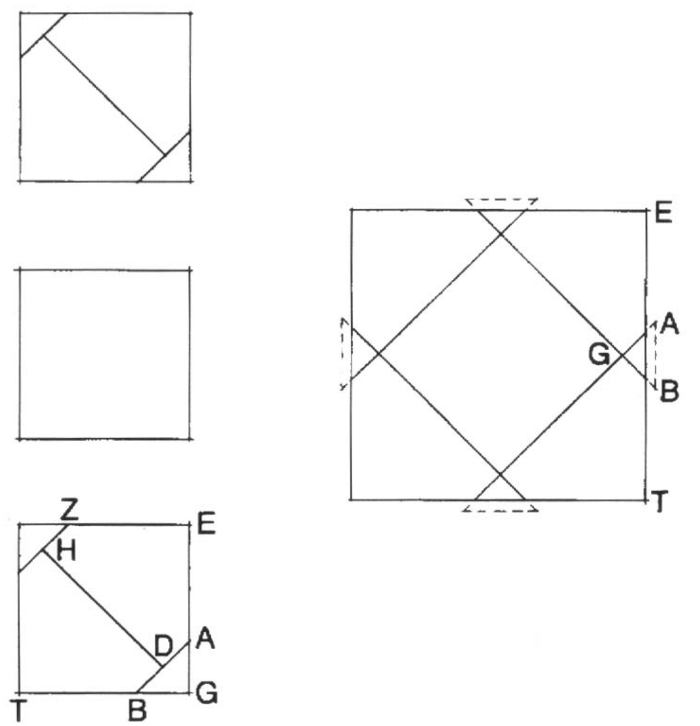


Figure 4. Abu ‘l-Wafa’’s demonstration disproving a solution proposed by a craftsman.

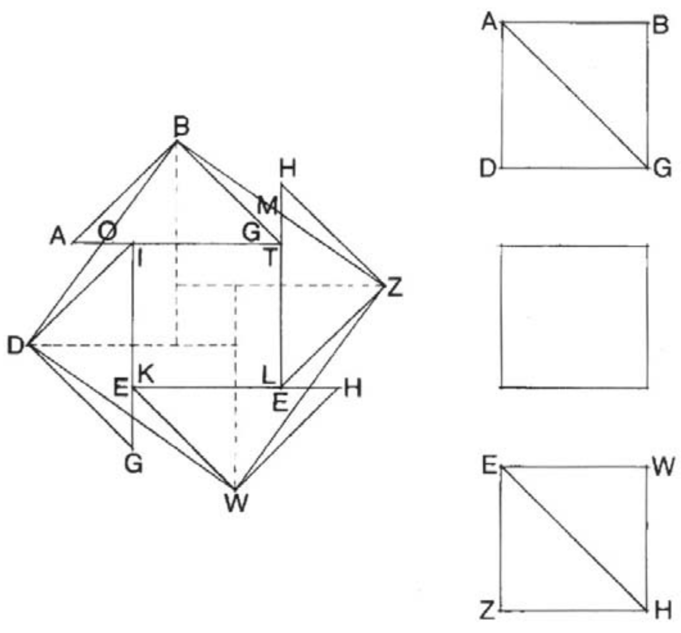


Figure 5. Abu ‘l-Wafa’’s solution to the problem of a square formed from three squares.

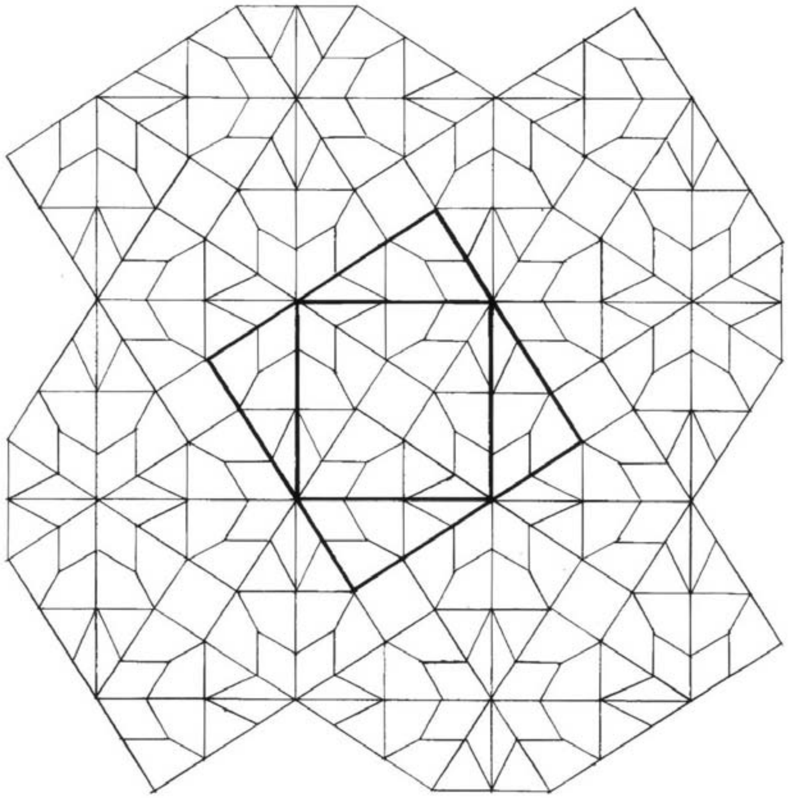


Figure 6. A special case of Abu ‘l-Wafa’’s solution for a square formed from five squares given in *On Interlocks of Similar or Corresponding Figures.*

Ozdural places the text, *On Interlocks of Similar or Corresponding Figures,* in Tabriz from around the reigns of Ghazan Khan (1295-1304) or Oljeitu (1304-1316). Like Abu ‘l-Wafa’’s text this text was also created by a geometer with the objective of showing cut and paste methods of construction as an alternative to more theoretical constructions. The author is mainly concerned with the transformation of one polygon into another. He shows how two decagons and a pentagonal star may be cut and rearranged to form a large decagon. He then places the resultant figure into a larger surface pattern (figs. 7). He also shows two methods of constructing a square from a rectangle of any proportions. By extension this permits an elegant solution to the problem of a large square from three smaller squares (figs. 8). The text also presents a puzzle-like construction of which

Ozdural says is “incoherent and inconsistent and can hardly be the work of a mathematician. The problem seems to have arisen as a challenge to the practically minded artisans” (fig. 9).[10]

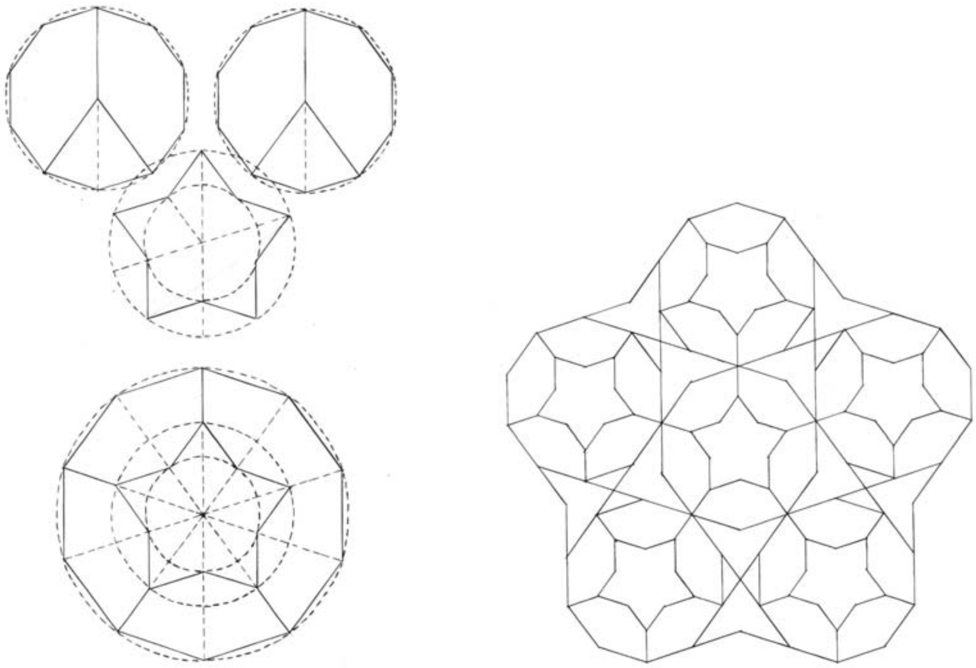


Figure 7. Designs from *On Interlocks of Similar or Corresponding Figures.*

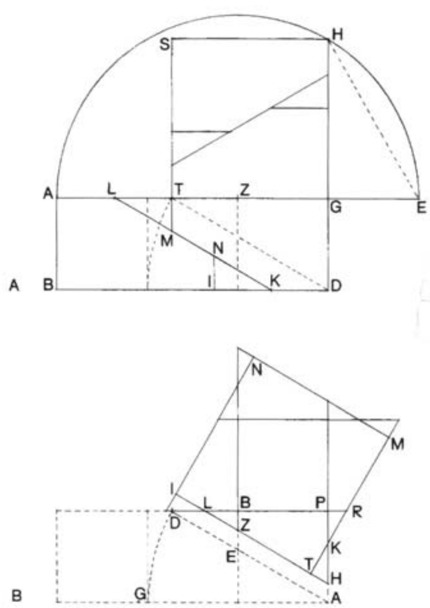


Figure 8. Was to form a square from a rectangle from the book *On Interlocks of Similar or Corresponding Figures.*

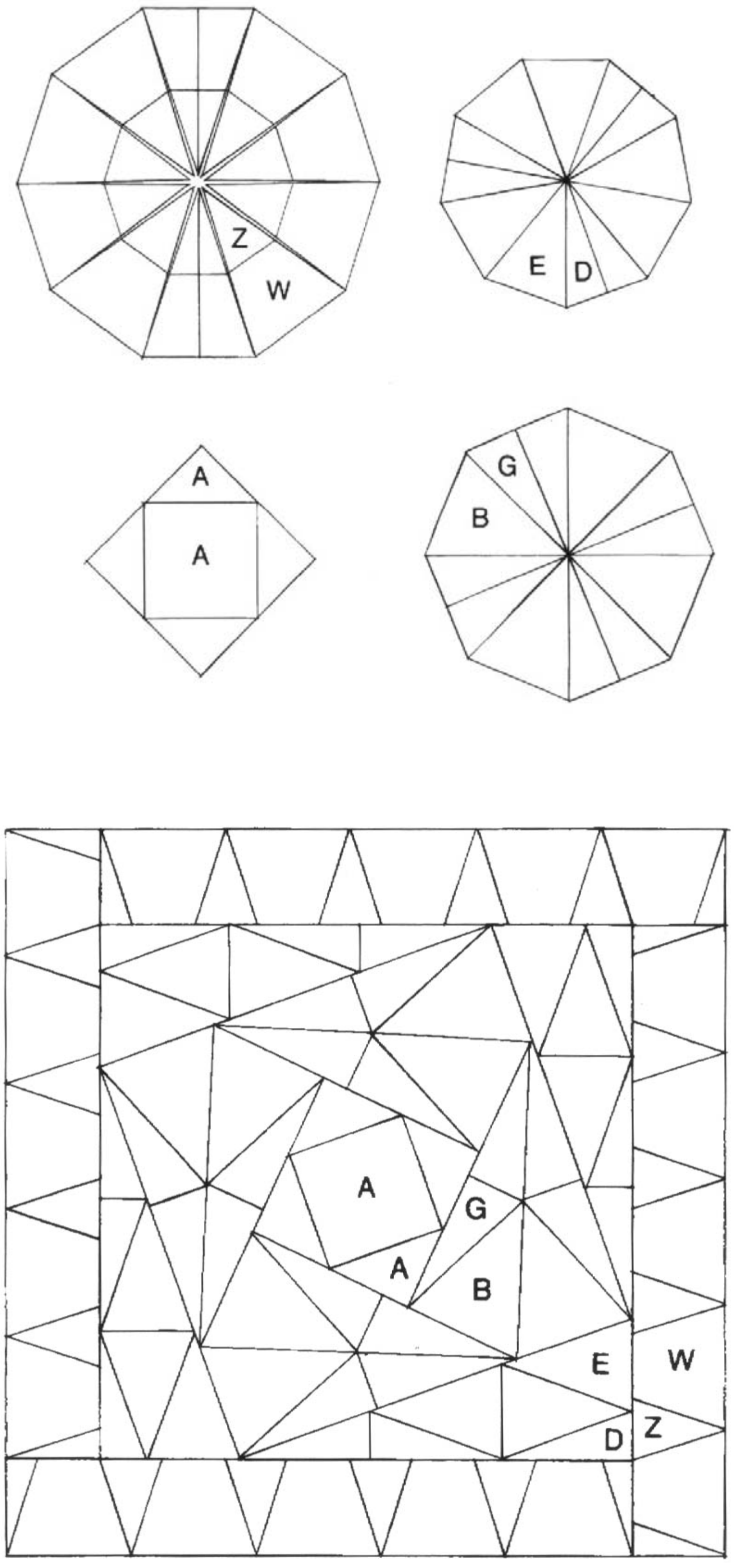


Figure 9. A pattern formed from polygons given in the text *On Interlocks of Similar or Corresponding Figures.*

Pattern scrolls

In contrast to the impression created by the texts discussed by Alpay Ozdural, there are scrolls that create a different impression of the expertise of architects and artisans. These seem to be pattern books created by and for architects and craftsmen for the design and ornamentation of buildings. They are mostly practical rather than theoretical works, and those that contain plans seem to be showing idealized types rather than plans for specific buildings. The fact that the scrolls often combine patterns for two- and three-dimensional architectural revetments, together with grid-based ground plans, suggests that they were compiled by master builders responsible for coordinating all aspects of the building project. Furthermore, Bulatov, a Russian writer on Islamic geometry, notes that by combining two- and three-dimensional designs within a single scroll helped to create an overall sense of unity within the buildings produced.

The Topkapi scroll, which Gulru Necipoglu dates to the late 15th or early 16th century,[11] is the best preserved of its kind and contains patterns for walls and vaults but no ground plans. Unfortunately there are no comparable scrolls from earlier periods. Necipoglu notes that there are no known pre-Mongol working drawings in any format but there are textual references to plans. One surviving fragment is an early sketch for an Ilkhanid muqarnas vault that had been found carved in a plaster slab at Takht-I Sulayman, site of a Mongol palace built for the Ilkhanid ruler Abaqa Khan in the 1270s. References are more common after the Timurid era but there are few surviving examples. Some Ottoman plans have been found drawn on squared paper, the earliest examples of which date from the late 15th and 16th centuries. Necipoglu believes the use of grid-based ground plans in Ottoman, Mughal, and Uzbek architecture probably has Timurid-Turkmen origins.[12]

The fragmentary Tashkent scrolls, attributed to an Uzbek master builder from Bukhara in the 16th century, are probably typical of pattern book scrolls. These contain geometric patterns and inscriptions intended for banna'i brick masonry, some ground plans on squared grids, and projections of vault designs – both muqarnas vaults and intersecting arch-net vaults forming stellate patterns. Necipoglu considers these scrolls to be workshop catalogs prepared to preserve the memory of ideal two- and three-dimensional patterns used by the workshop. They are based on the principle of a repeat unit – fragments of patterns meant to be multiplied or rotated by symmetry. The vault patterns are in red and black ink, further enhanced by color coding with orange, yellow and green pigments that would have helped those that understood the system to project the patterns into three dimensions. Without prior knowledge one cannot tell how a muqarnas vault plan can be projected into three dimensions. This ambiguity in the vault plans may have been deliberate, as a way of protecting craft knowledge. Mohammad Al-Asad has prepared a computer projection of a muqarnas plan but his results are largely hypothetical (figs. 10).[13] Necipoglu finds the designs in the Tashkent scrolls to be consistent with Uzbeck monuments preserved in Central Asia and notes that some scholars have found they use a proportional system of *girihs* (Persian, "knot") that reflect a simplification from the geometry used in Timurid and Uzbek monuments.[14]

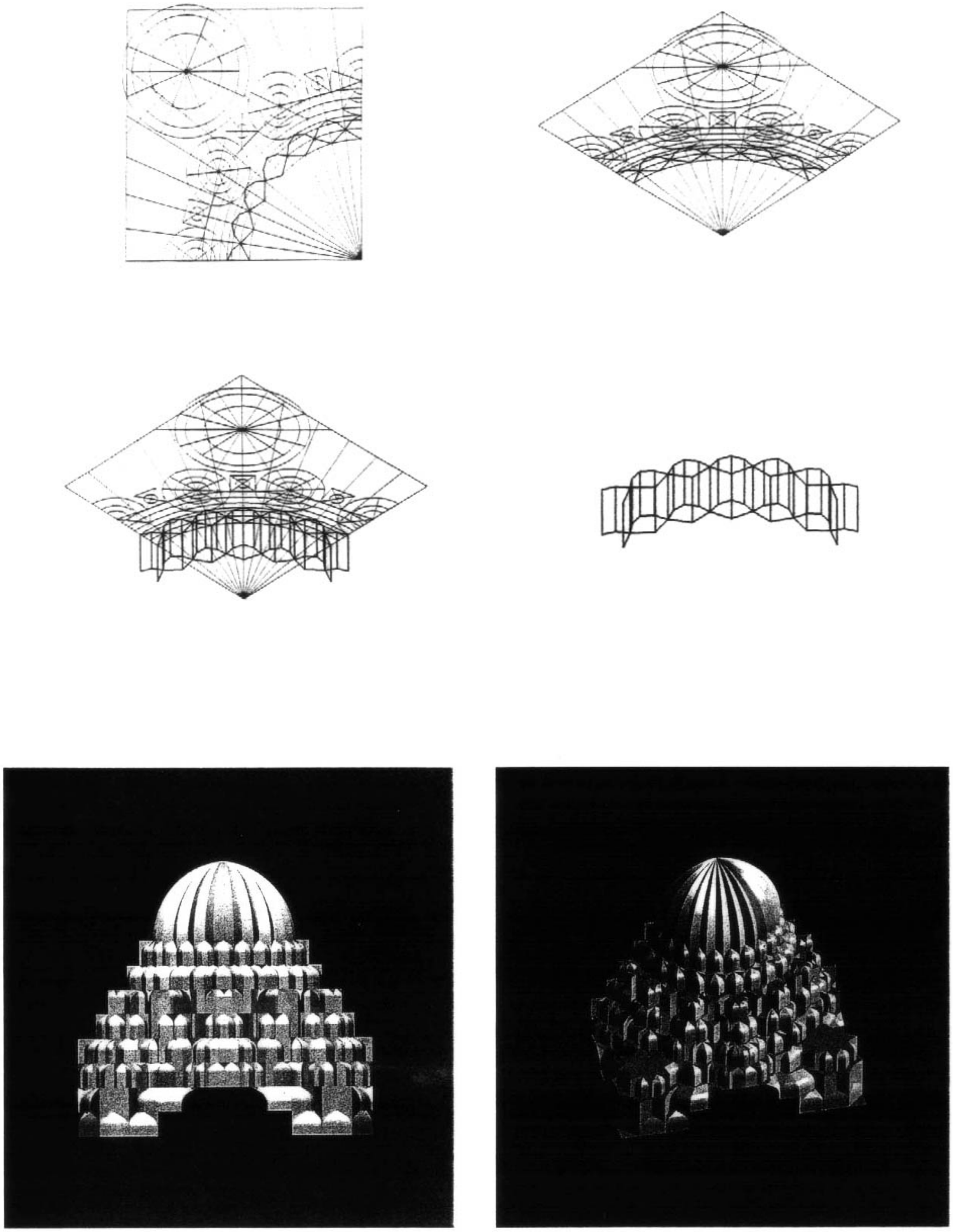


Figure 10. Mohammad Al-Asad’s computer generated solution for projecting a muquarnas vault from a 2D diagram.

Bulatov identified six basic grid systems in the geometric decoration of Central Asian monuments between the 10th and 12th centuries (square; square and its derivatives; semisquare and its derivatives, or double square; equilateral triangle and its derivatives; combinations of equilateral triangle and square; and the radial grid). Among these the radial and the simpler grids based on the square and the triangle were particular favorites of the Timurid period. Bulatov notes that radial symmetries were used in both two- and three-dimensional designs, by both Timurid and post-Timurid designers, as a way of harmonizing the overall scheme. In the Tashkent scrolls the most common grid systems were the square or 45-degree rotated square and the radial grid. The radially arranged tiers of the Tashkent scroll's muqarnas designs were intended for plaster vaults and appear to be elaborations on earlier radial muqarnas vaults used in the Shah-I Zinda complex at Samarqand.

The Topkapi scroll is a typical pattern scroll like the Tashkent scrolls but is unusually long – 29.5 meters – not a practical length for regular workshop use. It contains no date, writing, or watermark to indicate how it was put together or ended up in an Ottoman treasury. It appears to have been made from two or more shorter scrolls but the same graphic conventions and high quality paper is used thoughout leaving little doubt it is part of a consistent collection, possible all from the same hand. Necipoglu notes that the scroll's two-dimensional patterns were largely intended for brick and tile architecture rather than the Ottoman tradition of stone masonry. Like the Tashkent scrolls it contains geometric patterns and inscriptions on grids for banna'i masonry, two-dimensional star-and-polygon patterns, projections for radial muqarnas and stellate arch-net vaults, and details for architectural ornaments, but no building plans (figs. 11, 12, 13). According to Necipoglu, "These drawings, which would have had little relevance in the Ottoman context, constitute the largest known repertory of two- and three-dimensional geometrical patterns for Timurid-Turkmen or Early Safavid architectural revetments."[15] They were probably taken from Tabriz in 1474 or 1514 during one of several times the Ottomans conquered that city. Or they may have belonged to Timurid-Turkmen decorators invited (or drafted) to the court of Mehmed II in the 1470s. These drawings seem to represent a catalog of ideal types represented by few surviving examples. If these designs were once employed in Turkmen or early Safavid courts in Tabriz, they represent the only surviving record of a now lost architectural tradition.

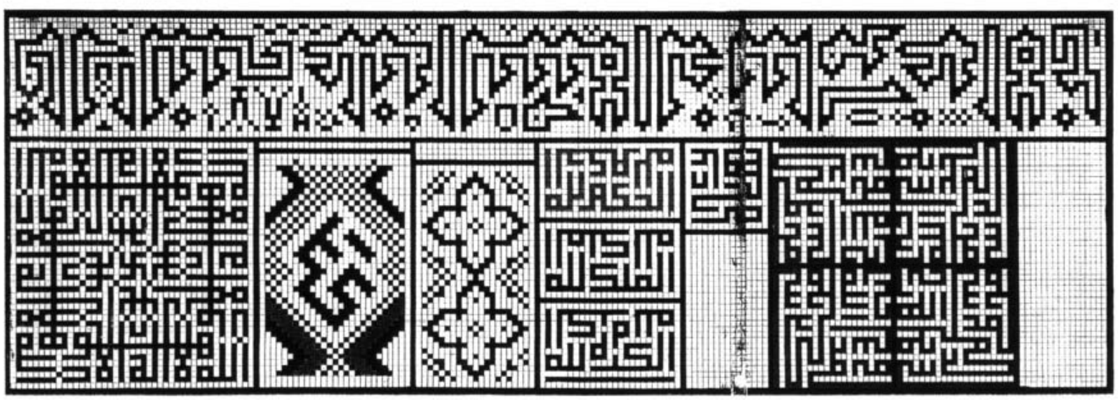


Figure 11. Geometric patterns and inscriptions for banna'i brick masonry from the Topkapi scroll.

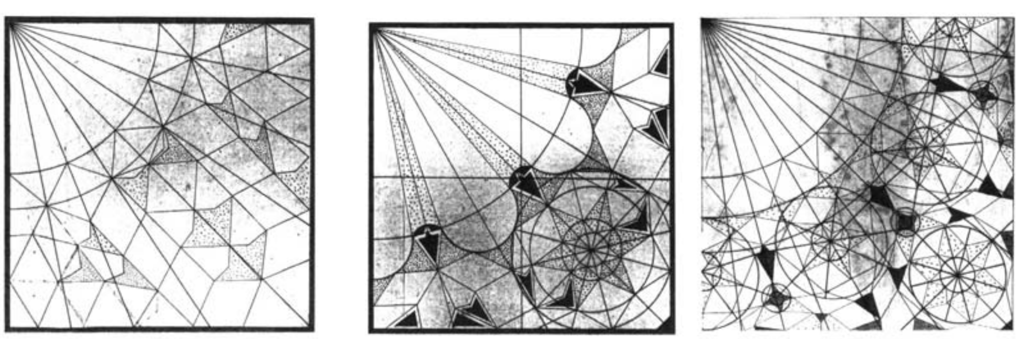


Figure 12. Muquarnas designs from the Topkapi scroll.

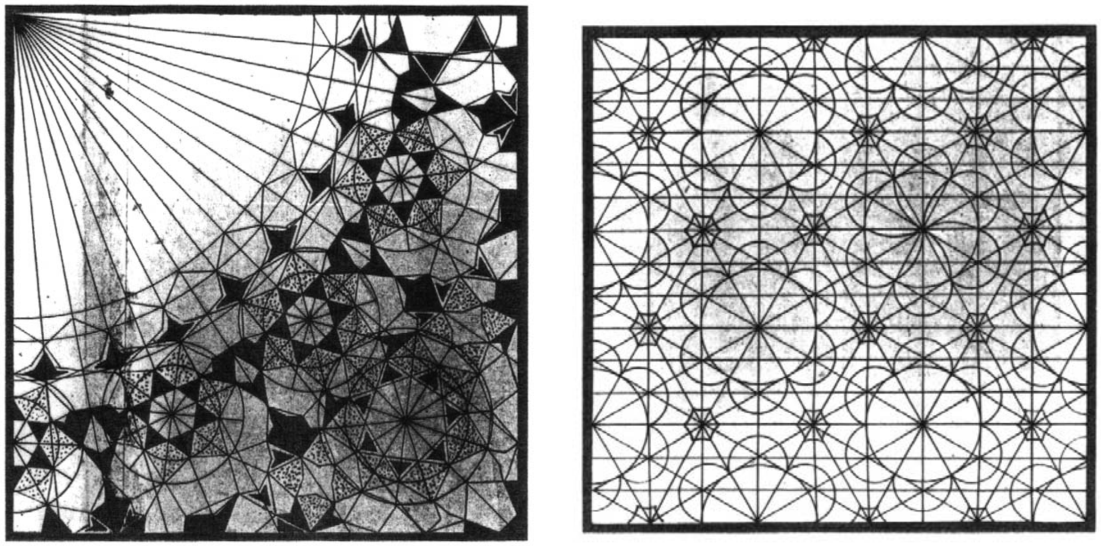


Figure 13. Muquarnas and wall designs from the Topkapi scroll.

There are also scroll traditions that have made it into the modern age. Caspar Purdon Clarke, an architect and resident of the British Embassy in Iran, acquired a collection of “roll books” most of which are now in the Library of the South Kensington Museum.[16] One identified as “Roll of Standard Patterns for Tessellated Work” contains two-dimensional geometric patterns which could be applied in several media. The second one identified as “Roll of Standard Patterns for Groined Vaulting” gives projections for stellate arch-net vaults. Other more fragile scrolls were cut up and mounted on cardboard and contain designs for the construction of arch curves, various ornamental details, vault projections (muqarnas and arch-net), square and hexagonal tile work patterns, geometric patterns and inscriptions for banna’i brick masonry, geometric window lattice patterns, and ground plans on square grids (as well as various sketches of fairies, birds, animals, mythical creatures, flowers, cypress trees, and vegetal patterns). Again, the objectives of these books appear designed to record workshop practice. Similar scrolls have been found in modern Iraq where practicing master builders use them as repositories of inherited craft secrets. Similar pattern scrolls have also been found in Morocco although these, like the Topkapi scroll, include no ground plans. The contemporary Moroccan engineer traces ground plans out directly on the ground, perhaps in an effort to maintain an element of secrecy.

Necipoglu notes, "The juxtaposition of planar and spatial geometric patterns in Islamic scrolls provides further evidence that the two were regarded as complementary systems."[17] Both interlocking geometric surface revetments and designs for muqarnas vaults are based on similar *girih* systems. These related patterns were often juxtaposed in both the scrolls as well as on buildings. This "supports Bulatov's observation that direct translatability between two- and three-dimensional geometric patterns imbued Islamic architecture in Central Asia with a harmonious sense of unity, synchronizing the proportional systems of ground plans, surface revetments, and decorative vaulting."[18] Actually, architectural plans and elevations, and the synchronization of their proportional systems, are a bit more problematical. Pattern scrolls rarely show ground plans and those that do use a simple square grid system that does not reveal the underlying geometric system employed.

<u>Studies from monuments</u>

Bulatov has done considerable work on the study of geometry in the plans and elevations of Islamic architecture in *Geometricheskaya Garmonizatsiya v Arkhitekture Srednei Azii IX-XVvv.*[19] Here Bulatov presents studies of plans, elevations, arches and ornamental patterns through over 250 illustrations. This is a work in Russian and not available in

English translation but fortunately Golombeck and Wilber, in chapter seven of *The Timurid Architecture of Iran and Turan*, have presented a summary of the highlights of Bulatov's work (itself built upon the contributions of a large number of Soviet scholars).[20]

Bulatov sees Islamic architectural plans as incorporating two processes simultaneously: one analytical (based on the use of grids), and the other geometric. The grids are based on the *gaz*, or cubit, of approximately two feet long. The system of geometric proportions generates irrational numbers based on the square, double square, equal lateral triangle, and pentagon. For the grid, one dimension within the plan such as the thickness of walls would establish the module for the plan. The geometric proportions were based on what Bulatov identifies as a generative unit. If a large domed chamber was planned, the length of its side would be the generative unit. Relationships to other rooms in the plan, elements of the façade, or the height of an iwan, would all be proportional to the generative unit. Timurid architects used essentially four related systems of proportion, or sets of ratios, which Bulatov notes had been and use since the 10th century (fig. 13, 14). The so-called "golden rectangle" does occur but seems to have been given no more significance than any other rectangle. The comment is also made that the geometric system employed was "not comparable to Western notions of proportion, which are concerned with the repetition of similar or related form,"[21] but this is not correct. Tons Brunes has done some comparable analyses of Egyptian, Greek, Roman, and Medieval monuments that show the same type of geometry being used[22]. Where Brunes gets into trouble is attributing this continuity of geometry to a secret esoteric tradition.

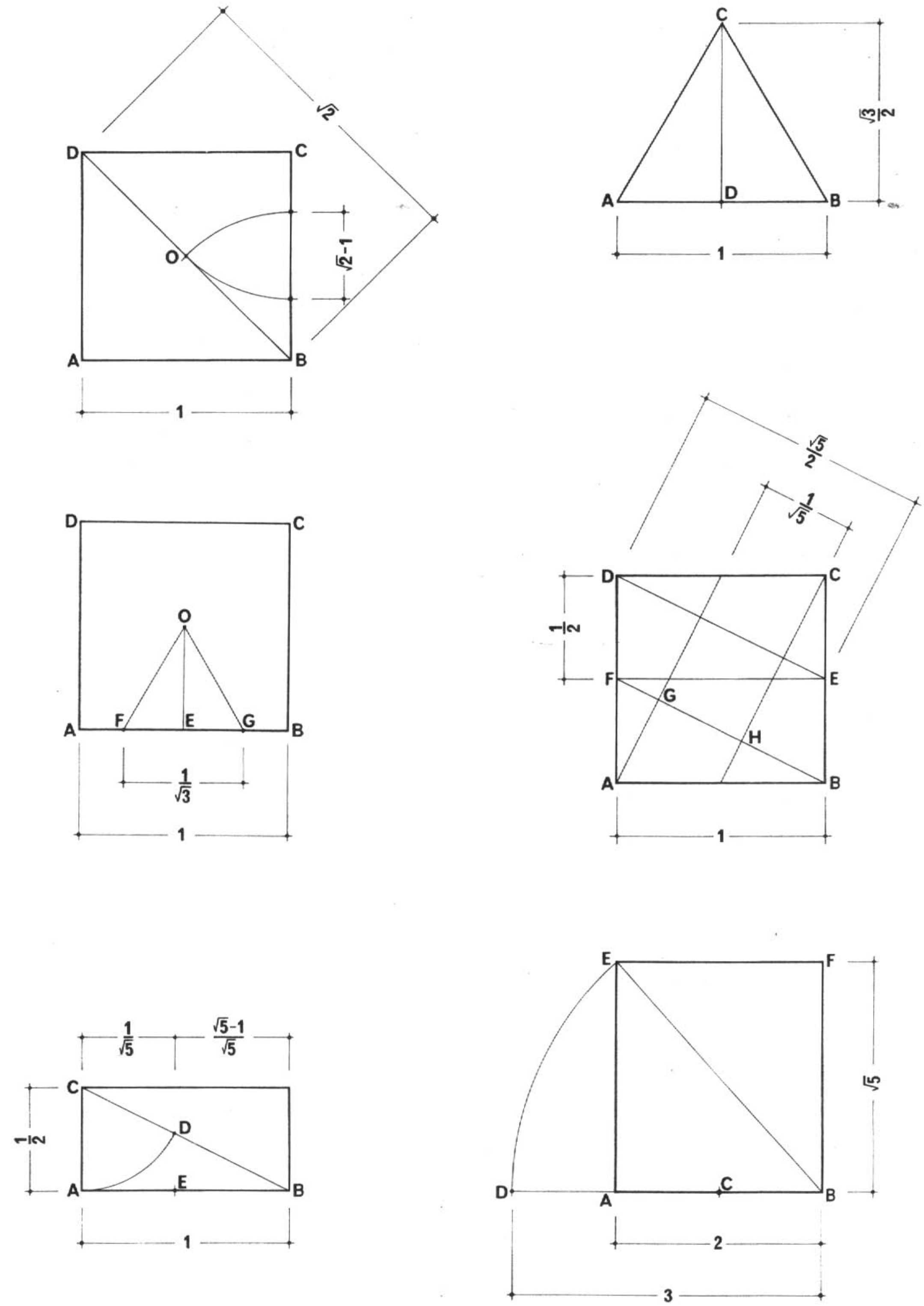


Figure 13. Bulatov’s summary of proportions used in architecture.

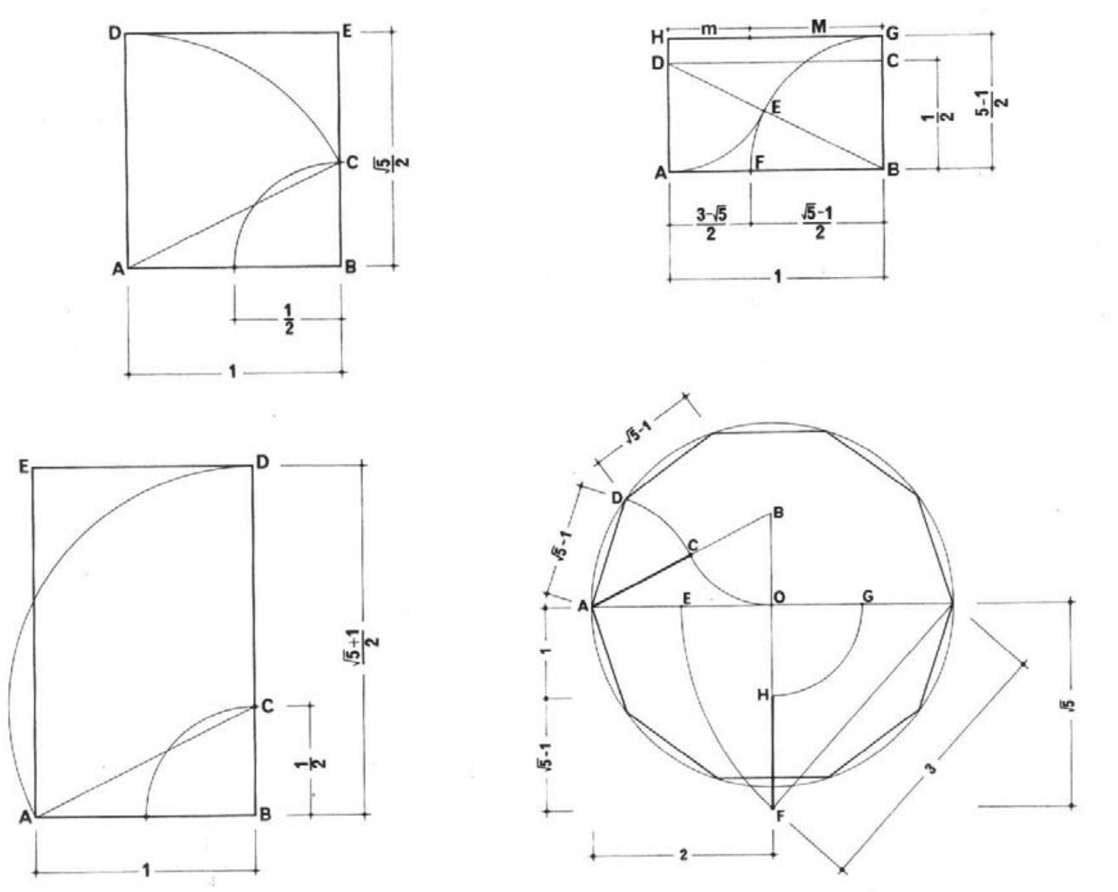


Figure 14. Bulatov's summary of proportions used in architecture.

Bulatov's drawings of these proportions are deceptively complex. For example, those proportions related to the square, and the proportion 1:√2, can be generated with one of several different diagrams (fig. 15). We have no way of knowing which diagram, or diagrams, Islamic architecture regularly used when designing a building. That they did use, and thought in terms of, some sort of diagram that could be easily constructed seems far more likely than the use of irrational numbers indicated in Bulatov's illustrations. What Bulatov has done is create the impression that the underlying system is more complicated than it really is. The fact is, none of these proportions are particularly sophisticated and any architect or craftsman with a simple compass and straightedge can easily discover them if motivated. How they are applied is the part that is culture bound.

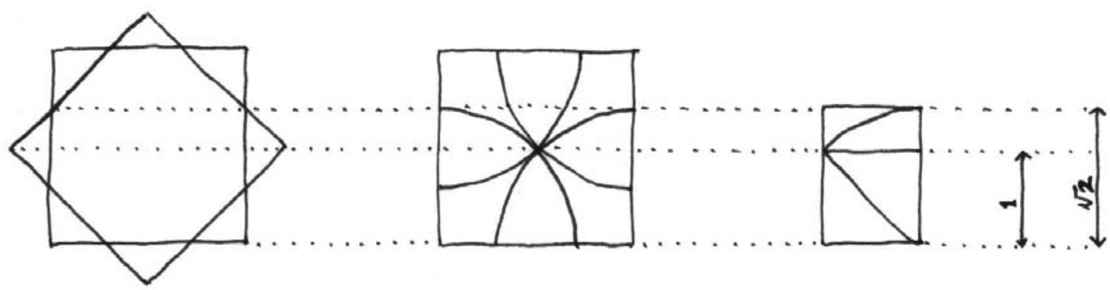
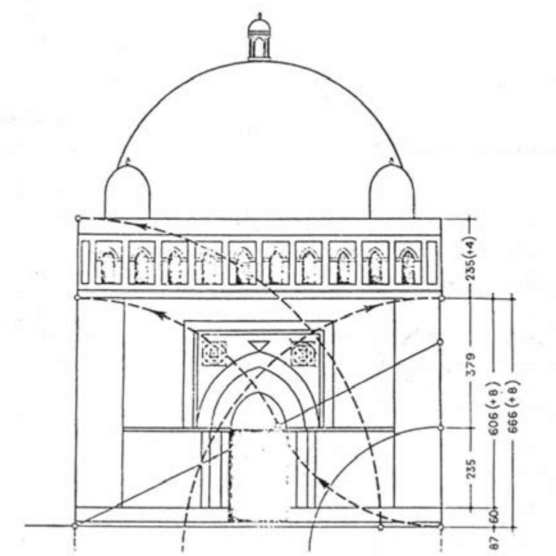
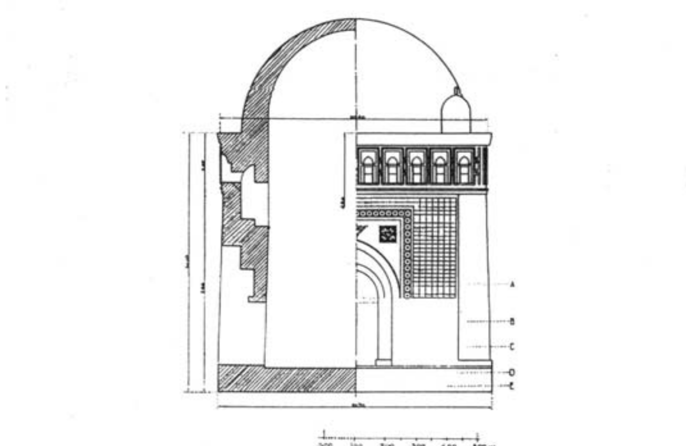


Figure 15. Several diagrams for producing the same proportion.

On the whole Bulatov's studies of Islamic monuments are quite good but there are some noteworthy criticisms that need to be addressed. Bernard O'Kane questions the accuracy of the plans with which Bulatov and others are using for their studies.[23] Further, he points to a case where Bulatov fails to take into account the inward slope of the building in the analysis of a façade (fig. 16). O'Kane is also of the opinion that there are so many systems of proportion available that Bulatov and others can simply keep searching until something fits. (As if they were merely seeing faces in clouds). To make his point O'Kane presents three studies of the shrine of Ahmad Yasavi in Turkistan – one by Bulatov and one each by Man'kovskaia and Zakhidov (figs. 17, 18, 19). O'Kane is of the opinion that if one of these is correct the others must be wrong. This misses the point. If the plans these scholars are working with are accurate then all their studies have some validity. Some of them focus on the overall proportions, while others focus on the proportions of particular rooms. Plus, this monument is unusually sophisticated with a variety of dome and vault types and geometry operating on several levels simultaneously. While we have no way of knowing what particular geometric devices the designers of this shrine employed, all three diagrams reveal something of the proportions that resulted. Furthermore, Bulatov has shown through studies of over seventy plans and sixty elevations of less complicated monuments, that geometric proportions are repeatedly applied with far too much precision and grace to be the result of forcing what is really a limited number of proportions to fit the monuments (if he is working from accurate surveys of the monuments[24]).



a. Bukhara, Mausoleum of the Samanids, 10th century, elevation (**after Bulatov**).



b. Bukhara, Mausoleum of the Samanids, 10th century, elevation (**after Rempel**, "The Mausoleum of Isma'il the Samanid", *Bulletin of the American Institute for Persian Art and Archaeology*, IV/4 [1936]).

Figure 16. Bulatov's and Rempel's designs of the Mausoleum of the Samanids, Bukhara.

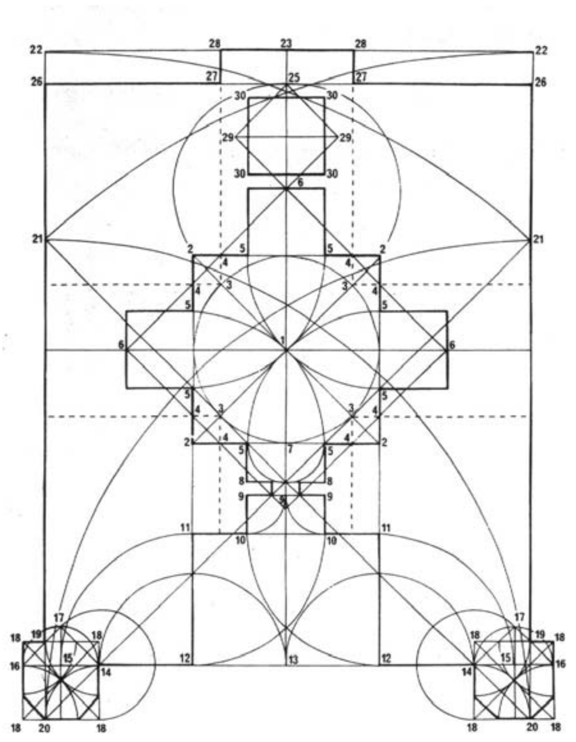


Figure 17. Turkistan, Shrine of Ahmad Yasavi, 1397-1399, after Man’kovskaia.

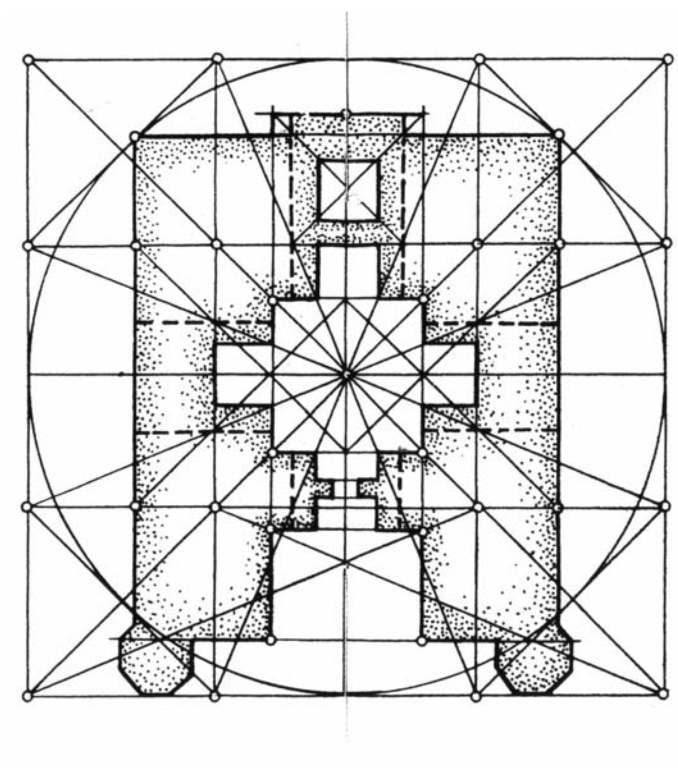


Figure 18. Turkistan, Shrine of Ahmad Yasavi, 1397-1399, after Zakhidov.

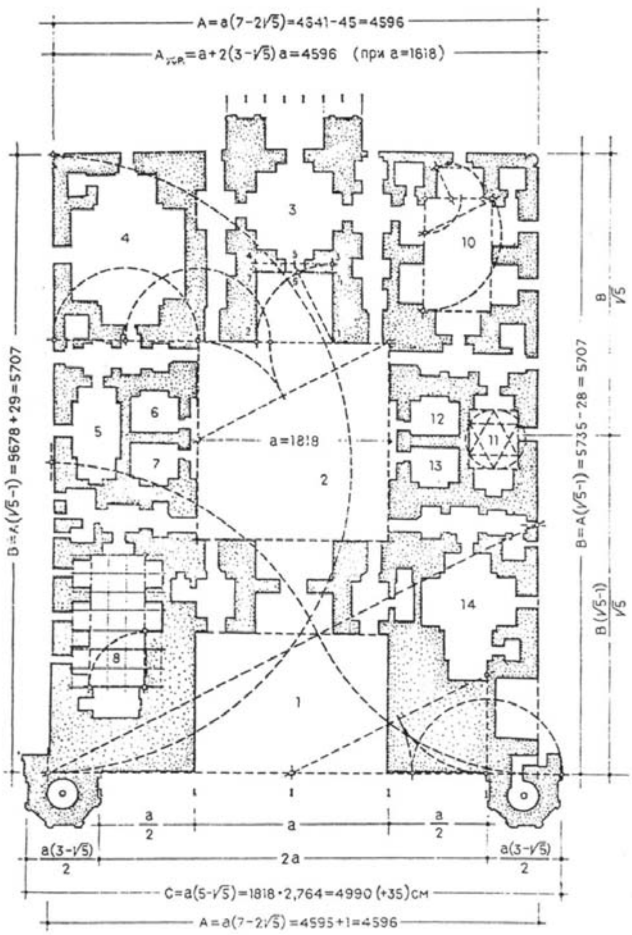


Figure 19. Turkistan, Shrine of Ahmad Yasavi, 1397-1399, after Bulatov.

Conclusion

From texts such as that written by Abu ‘l-Wafa, pattern books such as the Topkapi scroll, and from analysis of monuments, it is evident that geometry was an important factor in later Islamic architecture. What is less clear is what architects and craftsmen knew of geometry. Abu ‘l-Wafa’s text has suggested they knew very little, but pattern scrolls seem to have been the product of knowledgeable master builders documenting workshop knowledge of geometry. It may be that that what we are observing are two different categories of geometry. Abu ‘l-Wafa is a geometer who seems to be coming from a tradition that, like Euclid, requires proofs. Ground plans and the two- and three-dimensional geometry of the pattern scrolls use a class of non-Euclidean geometry that requires a compass and square but needs not proof.

Notes

[1] Issam El-Said, *Islamic Art and Architecture: The System of Geometric Design* (London: Garnet Publishing Limited, 1993).

[2] Keith Crichlow, *Order in Space: a Design Source Book* (New York: The Viking Press, 1969).

[3] Keith Crichlow, *Islamic Patterns: an Analytical and Cosmological Approach* (New York: Schocken Books, 1976).

[4] Lisa Golombek and Donald Wilber, *Timurid Architecture of Iran and Turan* (Princeton NJ: Princeton University Press, 1988), 137.

[5] Gulru Necipoglu, *The Topkapi Scroll – Geometry and Ornament in Islamic Architecture* (Santa Monica CA: The Getty Center for the History of Art and the Humanities, 1995).

[6] Alpay Ozdural, "Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World," Historia Mathematica 27 (2000), 171-201. This is largely an expansion of his earlier essay, "Omar Khayyam, Mathematicians, and *Conversazioni* with Artisans," Journal of the Society of Architectural Historians 57 (1995): 1, 54-71.

[7] The original of this text, composed in Arabic in thirteen chapters, no longer exists. Two versions are available: Istanbul, Aya Sofya 2753 and Uppsala, No. 324. In his "Mathematics and Arts" Ozdural reprints those portions of the Arabic text on which his paper is based.

[8] Ozdural, "Mathematics and Arts," 183.

[9] *Ibid*, 185.

[10] *Ibid*, 192.

[11] Necipoglu, "The Topkapi Scroll,"

[12] *Ibid*, 7.

[13] Ibid, 349-359.

[14] *Ibid*, 12,13.

[15] *Ibid*, 30.

[16] *Ibid*, 14.

[17] *Ibid*, 22.

[18] *Ibid*, 22.

[19] M. Gulatov, *Geometricheskaya Garmonizatsiya v Arkhitekture Srednei Azii IX-XVvv* (Moscow, 1978)

[20] Golombek, 137-173.

[21] Ibid, 140.

[22] Tons Brunes, The Secrets of Ancient Geometry – And its Use (Copenhagen: Rhodos International Science Publishers, 1967).

[23] Bernard O'Kane, "Poetry, Geometry and the Arabesque: Notes on Timurid Aesthetics," Annales Islamologiques Tome XXVI (1992), 64-78.

[24] This is no small "if." I tried my hand at a study of this plan but found that a photocopy from a book illustration lacked the scale or precision to be of any use. One can only trust that Bulatov and the others were working from sizable, accurate surveys of the monument.

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